聖公會林護紀念中學

2019-20 中一學位申請 第一階段面試 (數學解難) Set A

甲乙二人在玩一個遊戲：開始時，甲和乙分別有若干顆小石子，然後他們各自把自己的石子放在左手或右手中 (可以把石子全放在一隻手，而另一隻手沒有石子)。兩人皆知道自己及對方的石子總數，也可以控制自己放石子在左手或右手的數量，但不知道對方放石子在左手或右手的數量。

放好石子後，兩人把左手的石子拿出來，數算石子的數目。

如果甲的石子數目比乙多，或者一樣多，則甲贏得這一局。如果乙的石子數目比甲多，則乙贏得這一局。

然後第二局，再把右手的石子拿出來，數算石子的數目。決定勝負的方法與上一局一樣。如果甲兩局均勝出，則甲勝出遊戲。如果乙能從兩局中贏取最少一局，則乙勝出遊戲。

> 以下是一個例子：
> 假設甲有 10 顆石子，乙有 9 顆石子
> 甲在左手放了 7 顆，右手放了 3 顆。(共 10 顆)
> 乙在左手放了 5 顆，右手放了 4 顆。(共 9 顆)
> 第一局，二人比較左手的石子數目。甲的石子數目較多，所以甲贏得第一局。
> 第二局，二人比較右手的石子數目。乙的石子數目較多，所以乙贏得第二局。
> 由於乙能從兩局中贏取最少一局，所以乙勝出了遊戲。

1. 若甲有 6 顆石子，乙有 3 顆石子, 問甲的最佳策略是甚麼？(即他應該分別放多少顆石子在兩手中，令他勝出遊戲的機會最大？) 試簡單解釋原因。(2 分)

2. 若甲有 5 顆石子，乙有 3 顆石子, 問乙的最佳策略是甚麼？試簡單解釋原因。(2 分)

3. 當乙有 3 顆石子時, 若乙有必勝策略，則甲最多有多少顆石子？為甚麼？(2 分)

4. 小明說，當乙有 20 顆石子時，若甲和乙均使用最佳策略，則甲有 35 顆石子時，與甲只有 25 顆石子時相比，前者令甲的勝出機會較高。你同意嗎？試說明你的答案。(3 分)

(應考同學在面試作答時，不需重複讀問題。
另外，面試老師可能對你的解答作出追問)。

A and *B* are playing a game: In the beginning, *A* and *B* has certain numbers of pieces of small stones respectively. Then both of them put some of their own stones into their left hands or right hands (They may put all stones on one hand so that no stone on the other hand). Both *A* and *B* know the total number of stones that each player has, and both players can control the number of stones to be put in their own left or right hands. However, both players do not know how the other player distribute the stones in their left or right hands.

After the stone are put, both of them open their left hands and count their numbers of stones. If *A* has more stones than *B*, or if their numbers of stones are equal, then *A* wins this set. If *B* has more stones than *A*, then *B* wins this set.

Then in the next set, they compare the numbers of stones in the right hand. The rules to determine the winner of this set is the same as that in last set.
If *A* wins both sets, then *A* wins the game. If *B* wins at least one of the two sets, then *B* wins the game.

> Here is an example:
> Let *A* has 10 stones and *B* has 9 stones.
> *A* put 7 stones on left hand and 3 stones on right hand (so 10 stones in total).
> *B* put 5 stones on left hand and 4 stones on right hand (so 9 stones in total).
> In first set, comparing the stones on left hands, *A* got more stones so *A* won the set.
> In second set, comparing the stones on right hands. *B* got more stones so *B* won the set.
> As *B* won on at least one of the two sets, *B* won the game.

1. If *A* has 6 stones and *B* has 3 stones, what is the best strategy for *A*? (i.e. How should he distribute the number of stones in his hands so that he will have the highest chance to win the game?) Please briefly explain. (2 marks)

2. If *A* has 5 stones and *B* has 3 stones, what is the best strategy for *B*? Briefly explain. (2 marks).

3. When *B* has 3 stones, if *B* has a strategy to guarantee to win, what is the maximum number of stones that *A* has? Why? (2 marks)

4. Ming said, when *B* has 20 stones, if both *A* and *B* are using their best strategies, then *A* will have a better chance to win when *A* has 35 stones then that when *A* has 25 stones. Do you agree? Explain your answer. (3 marks)

      (In the interview, candidates need not repeat the questions to the interviewers).
      (The interviewers may ask you further question to elaborate your answer).